Diploma Programme

Mathematics mind map
Analysis and approaches SL

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Mathematics mind map

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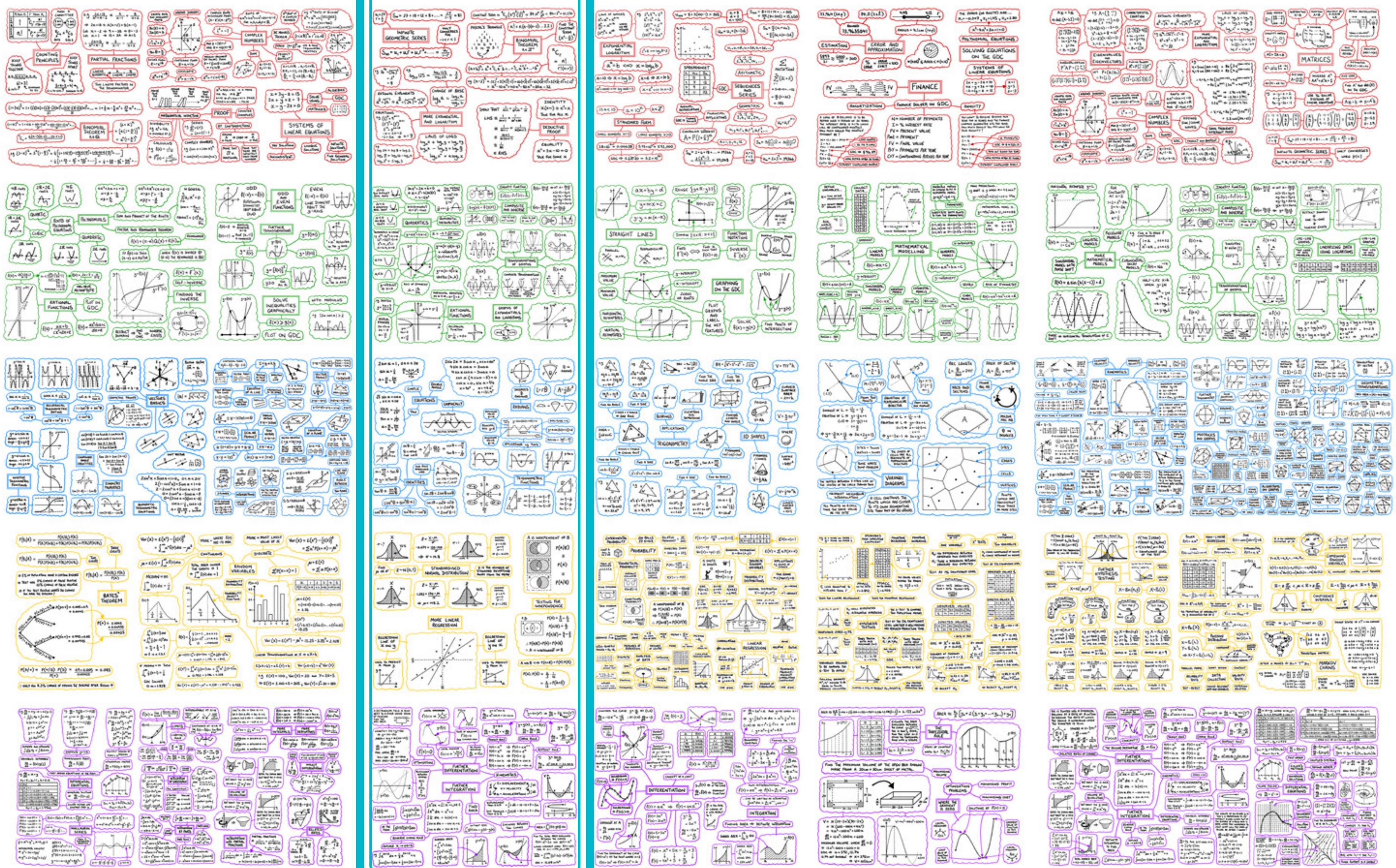
	Analysis and approaches AHL	Analysis and approaches SL	Common content	Applications and interpretation SL	Applications and interpretation AHL
Number and algebra					
Functions					
Geometry and trigonometry					
Statistics and probability					
Calculus					



Number and algebra

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INFINITE GEOMETRIC SERIES

$S_\infty = u_1 + u_1 r + u_1 r^2 + \ldots = \frac{u_1}{1-r}$

ONLY CONVERGES FOR $-1 < r < 1$

$u_1 = 27$, $r = \frac{2}{3}$

$S_\infty = 27 + 18 + 12 + 8 + \ldots = \frac{27}{1-\frac{2}{3}} = 81$

BINOMIAL THEOREM

$n \in \mathbb{Z}^+$

PASCAL'S TRIANGLE

$$\begin{matrix} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \end{matrix}$$

${}^5C_3$

$n! = n(n-1)(n-2)\ldots 3.2.1$

FIND THE CONSTANT TERM $\left(x^2 - \frac{3}{x}\right)^9$

CONSTANT TERM $= {}^9C_3 (x^2)^3 \left(\frac{-3}{x}\right)^6 = 84 x^6 \cdot \frac{3^6}{x^6} = 84 \times 3^6 = 61,236$

${}^nC_r = \frac{n!}{r!(n-r)!}$

$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + \ldots + {}^nC_r a^{n-r} b^r + \ldots + b^n$

e.g. $(x-2)^5 = (x)^5 + 5(x)^4(-2)^1 + 10(x)^3(-2)^2 + 10(x)^2(-2)^3 + 5(x)^1(-2)^4 + (-2)^5$
$= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$

MORE EXPONENTIAL AND LOGARITHM

RATIONAL EXPONENTS

$a^{\frac{1}{m}} = \sqrt[m]{a} \Rightarrow a^{\frac{n}{m}} = (\sqrt[m]{a})^n = \sqrt[m]{a^n}$

e.g. $16^{3/4} = (\sqrt[4]{16})^3$
$= 2^3$
$= 8$

CHANGE OF BASE

$\log_b a = \frac{\log_c a}{\log_c b}$

e.g. $\log_{25} 125 = \frac{\log_5 125}{\log_5 25} = \frac{3}{2}$

EQUATIONS

e.g. $\left(\frac{1}{3}\right)^x = 9^{x+1}$
$(3^{-1})^x = (3^2)^{x+1}$
$3^{-x} = 3^{2x+2}$
$-x = 2x + 2$
$-2 = 3x \Rightarrow x = \frac{-2}{3}$

LAWS OF LOGS

$\log_a x + \log_a y = \log_a xy$
$\log_a x - \log_a y = \log_a \frac{x}{y}$
$\log_a x^n = n \log_a x$

e.g.
$\log 24 = \log 8 + \log 3$
$\log_3 \frac{10}{4} = \log_3 10 - \log_3 4$
$\log_4 3^5 = 5 \log_4 3$

DEDUCTIVE PROOF

IDENTITY

$x(x+1) \equiv x^2 + x$

TRUE FOR ALL x

EQUALITY

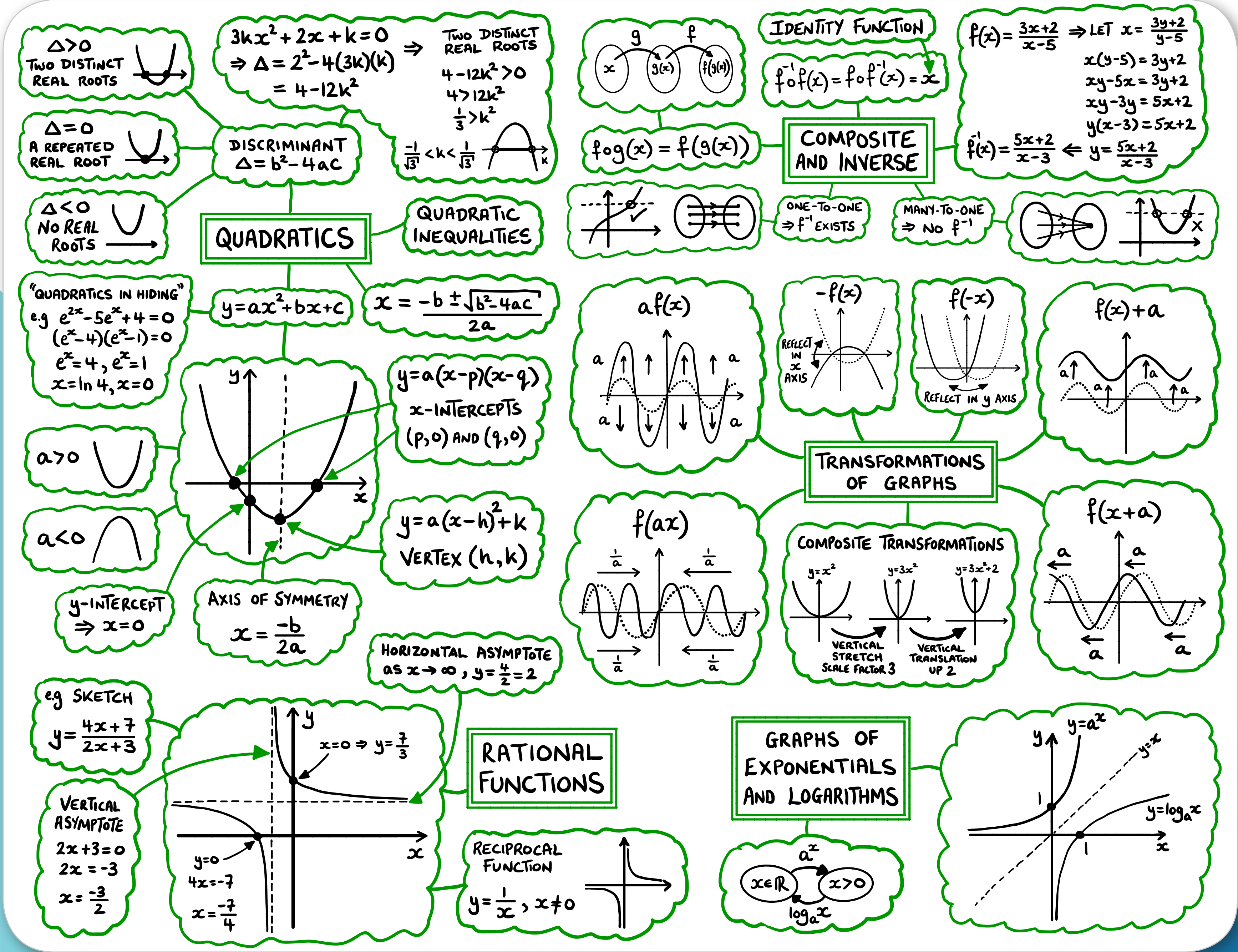
$x^2 + 3x - 10 = 0$

TRUE FOR SOME x

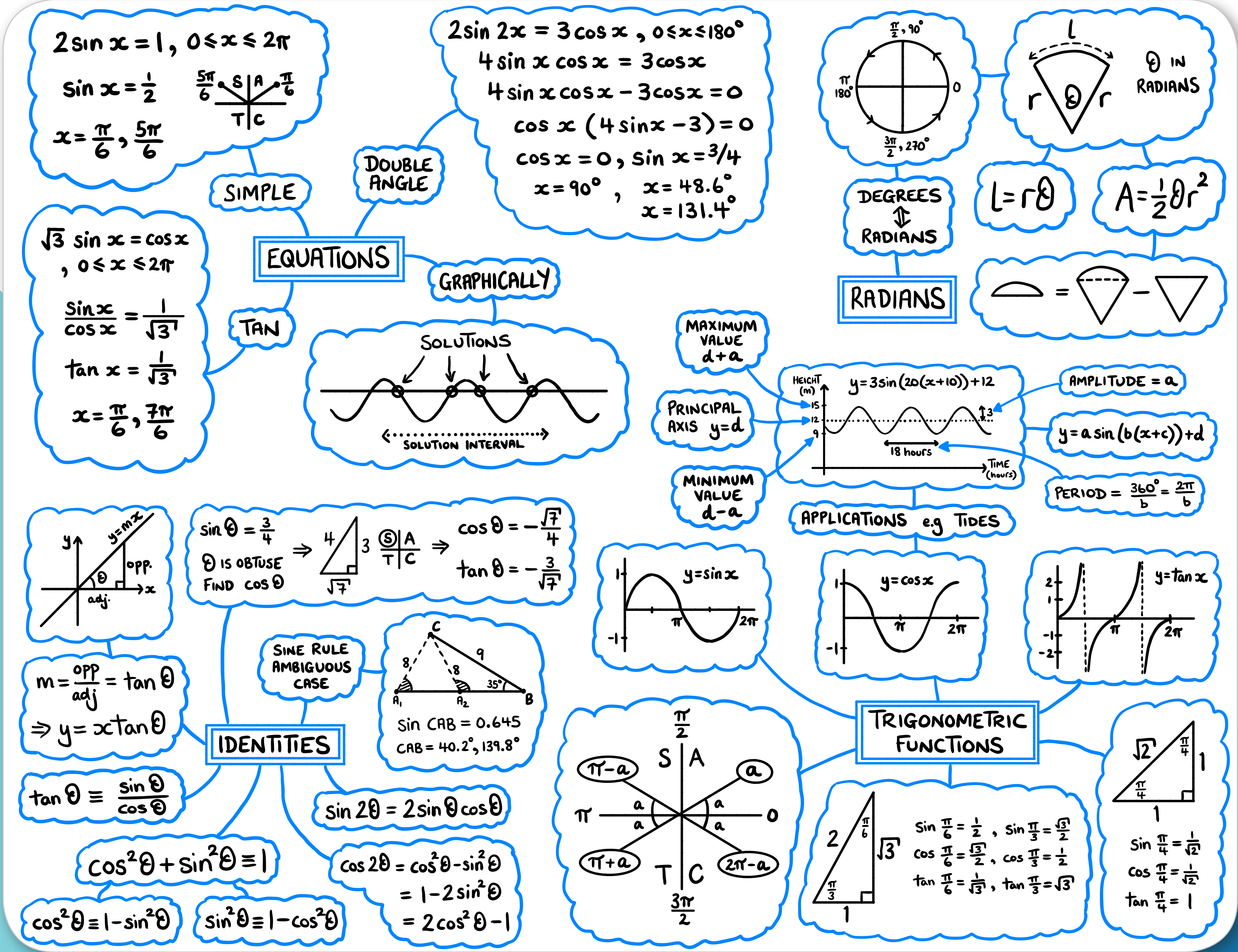
SHOW THAT $\frac{1}{m+1} + \frac{1}{m^2+m} = \frac{1}{m}$

$\text{LHS} \equiv \frac{1}{m+1} + \frac{1}{m^2+m}$
$\equiv \frac{1}{m+1} + \frac{1}{m(m+1)}$
$\equiv \frac{m+1}{m(m+1)}$
$\equiv \frac{1}{m}$
$\equiv \text{RHS}$

Functions


QUADRATICS
$\Delta > 0$ Two distinct real roots
$\Delta = 0$ A repeated real root
$\Delta < 0$ No real roots
DISCRIMINANT $\Delta = b^2 - 4ac$
$3kx^2 + 2x + k = 0 \Rightarrow$ Two distinct real roots
$\Rightarrow \Delta = 2^2 - 4(3k)(k)$
$= 4 - 12k^2$
$4 - 12k^2 > 0$
$4 > 12k^2$
$\frac{1}{3} > k^2$
$\frac{-1}{\sqrt{3}} < k < \frac{1}{\sqrt{3}}$
k
QUADRATIC INEQUALITIES
"QUADRATICS IN HIDING"
e.g $e^{2x} - 5e^x + 4 = 0$
$(e^x - 4)(e^x - 1) = 0$
$e^x = 4,\ e^x = 1$
$x = \ln 4,\ x = 0$
$y = ax^2 + bx + c$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$a > 0$
$a < 0$
y
x
$y = a(x-p)(x-q)$
x-intercepts
$(p,0)$ and $(q,0)$
$y = a(x-h)^2 + k$
Vertex (h,k)
y-intercept $\Rightarrow x = 0$
Axis of symmetry $x = \frac{-b}{2a}$
COMPOSITE AND INVERSE
g f
x g(x) f(g(x))
$f \circ g(x) = f(g(x))$
Identity function
$f^{-1} \circ f(x) = f \circ f^{-1}(x) = x$
$f(x) = \frac{3x+2}{x-5} \Rightarrow$ Let $x = \frac{3y+2}{y-5}$
$x(y-5) = 3y+2$
$xy - 5x = 3y + 2$
$xy - 3y = 5x + 2$
$y(x-3) = 5x + 2$
$f^{-1}(x) = \frac{5x+2}{x-3} \Leftarrow y = \frac{5x+2}{x-3}$
One-to-one $\Rightarrow f^{-1}$ exists
Many-to-one $\Rightarrow$ No f^{-1}
x
TRANSFORMATIONS OF GRAPHS
$af(x)$
a a a a
$-f(x)$
Reflect in x axis
$f(-x)$
Reflect in y axis
$f(x) + a$
a a a
$f(ax)$
$\frac{1}{a}$ $\frac{1}{a}$ $\frac{1}{a}$ $\frac{1}{a}$
COMPOSITE TRANSFORMATIONS
$y = x^2$
$y = 3x^2$
$y = 3x^2 + 2$
Vertical stretch scale factor 3
Vertical translation up 2
$f(x+a)$
a a a a
RATIONAL FUNCTIONS
e.g sketch $y = \frac{4x+7}{2x+3}$
Horizontal asymptote as $x \to \infty$, $y = \frac{4}{2} = 2$
$x = 0 \Rightarrow y = \frac{7}{3}$
y
x
$y = 0$
$4x = -7$
$x = \frac{-7}{4}$
Vertical asymptote
$2x + 3 = 0$
$2x = -3$
$x = \frac{-3}{2}$
Reciprocal function
$y = \frac{1}{x},\ x \neq 0$
GRAPHS OF EXPONENTIALS AND LOGARITHMS
a^x
$x \in \mathbb{R}$
$x > 0$
$\log_a x$
y
$y = a^x$
$y = x$
$y = \log_a x$
1
1
x

Geometry and trigonometry



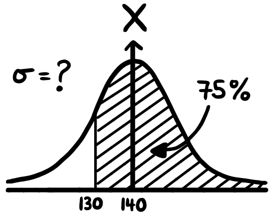
Statistics and probability

Standardised Normal Distribution

$Z \sim N(0,1)$

Z is the number of standard deviations away from the mean

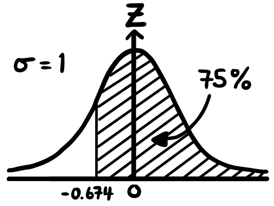
Find μ or σ



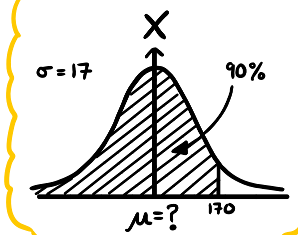
$Z = \frac{X-\mu}{\sigma}$

$-0.674 = \frac{130-140}{\sigma}$

$\Rightarrow \sigma = 14.8$



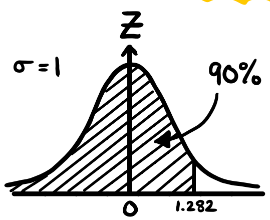
Inverse Normal
$p = 0.25$
$\mu = 0$
$\sigma = 1$
$\Rightarrow Z = -0.674$



$Z = \frac{X-\mu}{\sigma}$

$1.282 = \frac{170-\mu}{17}$

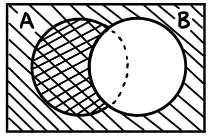
$\Rightarrow \mu = 148.2$



Inverse Normal
$p = 0.9$
$\mu = 0$
$\sigma = 1$
$\Rightarrow Z = 1.282$

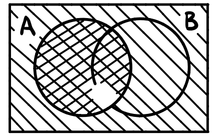
Testing for Independence

A is independent of B



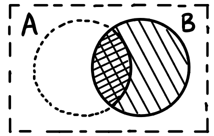
$P(A|B')$

$=$



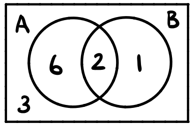
$P(A)$

$=$



$P(A|B)$

e.g.



$P(A) = \frac{8}{12} = \frac{2}{3}$

$P(A|B) = \frac{2}{3}$

$P(A|B') = \frac{6}{9} = \frac{2}{3}$

$\therefore P(A|B') = P(A) = P(A|B)$

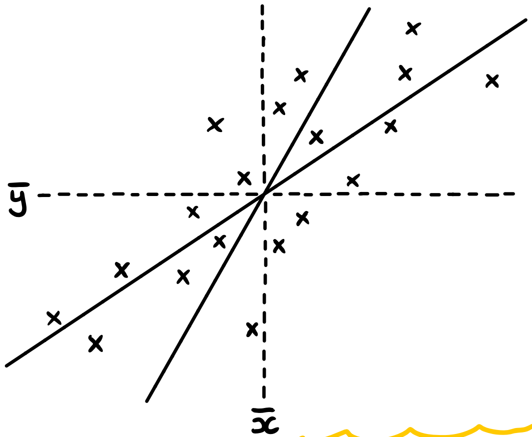
$\therefore$ A is independent of B

A ind. B $\Leftrightarrow P(A \cap B) = P(A).P(B)$

$P(A).P(B) = \frac{2}{3} \cdot \frac{1}{4}$

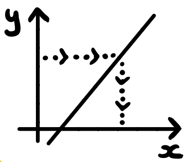
$= \frac{2}{12} = P(A \cap B)$

More Linear Regression



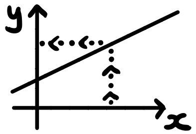
Regression line of x on y

Used to predict x from y



Regression line of y on x

Used to predict y from x



Calculus

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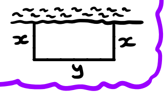
FURTHER DIFFERENTIATION

OPTIMISATION

A RECTANGULAR FIELD IS BUILT NEXT TO A RIVER USING 120m OF FENCING. FIND THE MAX. AREA.



LENGTH $= 2x + y = 120$

$\Rightarrow y = 120 - 2x$

AREA $= xy = x(120-2x)$

$A = 120x - 2x^2$

$\frac{dA}{dx} = 120 - 4x$

MAX. WHEN $\frac{dA}{dx} = 0$

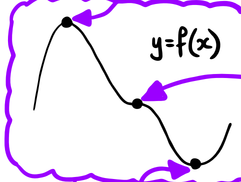
$4x = 120 \Rightarrow x = 30\text{m}$

MAX. AREA $= 30 \times 60 = 1800\text{ m}^2$

$f'(x) = 0$

LOCAL MAXIMUM

$f''(x) < 0 \Rightarrow$ (+ 0 −, $f'(x)$)



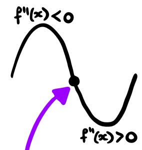
LOCAL MINIMUM

$f''(x) > 0 \Rightarrow$ (− 0 +, $f'(x)$)

POINTS OF INFLEXION

$f''(x) = 0$

AND CHANGES SIGN

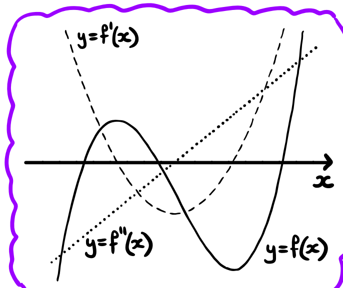


THE SECOND DERIVATIVE

$\frac{d^2y}{dx^2}$ OR $f''(x)$

RELATIONSHIP BETWEEN GRAPHS

$f(x)$ ↕ $f'(x)$ ↕ $f''(x)$



Standard derivatives

$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$

$f(x) = e^x \Rightarrow f'(x) = e^x$

$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$

$f(x) = \sin x \Rightarrow f'(x) = \cos x$

$f(x) = \cos x \Rightarrow f'(x) = -\sin x$

PRODUCT RULE

$y = uv$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

e.g. $y = x^2 \sin x$

$\frac{dy}{dx} = x^2 \cdot \cos x + 2x \cdot \sin x$

CHAIN RULE

$y = g(u)$, $u = f(x)$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

e.g. $y = \sin(x^2) \Rightarrow \frac{dy}{dx} = 2x \cdot \sin(x^2)$ (DIFF.)

QUOTIENT RULE

$y = \frac{u}{v}$

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

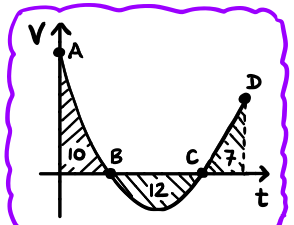
e.g. $y = \frac{\sin x}{x^2}$

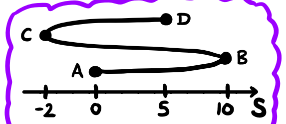
$\frac{dy}{dx} = \frac{x^2 \cdot \cos x - 2x \cdot \sin x}{x^4}$

KINEMATICS

$\frac{d}{dt}$: S = DISPLACEMENT → V = VELOCITY → a = ACCELERATION

$\int dt$: a = ACCELERATION → V = VELOCITY → S = DISPLACEMENT





DISPLACEMENT FROM A TO D $= \int v\,dt = 10 - 12 + 7 = 5\text{m}$

DIST. TRAVELLED FROM A TO D $= \int |v|\,dt = 10 + 12 + 7 = 29\text{m}$

FURTHER INTEGRATION

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\int e^x dx = e^x + c$

$\int \frac{1}{x} dx = \ln|x| + c$

$\int \sin x\, dx = -\cos x + c$

$\int \cos x\, dx = \sin x + c$

REVERSE CHAIN RULE

OF THE FORM $\int g'(x) f(g(x))\,dx$

$\int \frac{\sin x}{\cos x} dx = -\ln|\cos x| + c$

$\int 2x(x^2+1)^4 dx = \frac{1}{5}(x^2+1)^5 + c$

REPLACE $x \rightarrow ax+b$

e.g. $\int e^{3x+2} dx = \frac{1}{3}e^{3x+2} + c$

$\int \cos(2x-1)\,dx = \frac{1}{2}\sin(2x-1) + c$

FIND AREA

WITHOUT GDC

$\int_a^b g'(x)\,dx = g(b) - g(a)$

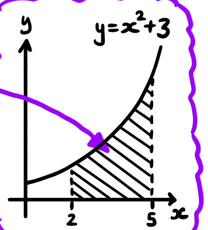
SHADED AREA

$= \int_2^5 x^2 + 3\,dx$

$= \left[\frac{1}{3}x^3 + 3x\right]_2^5$

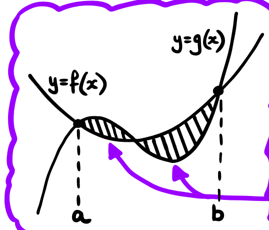
$= \left(\frac{125}{3} + 15\right) - \left(\frac{8}{3} + 6\right)$

$= 48 \text{ units}^2$



ENCLOSED BETWEEN TWO CURVES

AREA $= \int_a^b |f(x) - g(x)|\,dx$



FIND THE TOTAL AREA ENCLOSED BETWEEN THE CURVES $f(x) = x^3 - 5x$ AND $g(x) = x^2$

CURVES INTERSECT WHEN $f(x) = g(x)$

GDC $\Rightarrow x = -1.791, 0, 2.791$

TOTAL AREA $= \int_{-1.791}^{2.791} |(x^3 - 5x) - (x^2)|\,dx$

GDC $= 15.08 \text{ units}^2$